

AMPERE'S LAW :

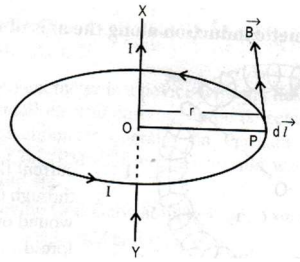
The line integral of magnetic field of induction \vec{B} around any closed path in free space is equal to absolute permeability of free space (μ_0) times the total current flowing through area bounded by the path.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

In Ampere's law, we imagine an Amperian loop i.e. a closed curve around a current carrying conductor. Further we imagine the loop (selected according to our convenience) to be made up of large number of small elements each of length dl . Then we determine the scalar products of \vec{B} and $d\vec{l}$ for each element and add all such products for entire loop. The direction along which loop is traced is the direction of element of length $d\vec{l}$
 $\oint \vec{B} \cdot d\vec{l} = \oint B \cdot dl \cos\theta$, where θ is the angle between \vec{B} and $d\vec{l}$

MAGNETIC FIELD DUE TO A LONG STRAIGHT CONDUCTOR CARRYING CURRENT:

Consider as infinitely long straight conductor XY carrying an electric current I . Let P be a point at a distance r from the conductor. We have to determine the magnetic induction of magnetic field at P due to current flowing through the conductor.



Let us choose an Amperian loop as an imaginary circle of radius r (perpendicular to straight conductor).

\vec{B} : magnetic induction at P , due to current I flowing through the conductor.

$d\vec{l}$: length of small element of circle around the wire

According to Ampere's law,

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$, but $\oint \vec{B} \cdot d\vec{l} = \oint B \cdot dl \cos\theta$, where θ is the angle between \vec{B} and $d\vec{l}$ which is zero. Thus,

$$\oint B \cdot dl = \mu_0 I \quad \text{Therefore, } B \oint dl = \mu_0 I$$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{4\pi} \frac{2I}{r}$$

Magnetic Induction along the axis of a long straight solenoid:

Let n : number of turns per unit length
 I : current sent through it, due to which magnetic field is created.

Consider a rectangular path ABCD. Let $AB=L$. Hence, nL is the number of turns enclosed by the rectangle ABCD.

Total current flowing = nLI

\vec{B} : Magnetic induction at a point well inside the solenoid.

According to Ampere's law, $\oint \vec{B} \cdot d\vec{l} = \mu_0(nLI)$. But for ABCDA,

$$\oint \vec{B} \cdot d\vec{l} = \int_A^B \vec{B} \cdot d\vec{l} + \int_B^C \vec{B} \cdot d\vec{l} + \int_C^D \vec{B} \cdot d\vec{l} + \int_D^A \vec{B} \cdot d\vec{l}$$

$$\int_B^C \vec{B} \cdot d\vec{l} = \int_C^D \vec{B} \cdot d\vec{l} = 0, \text{ since } \vec{B} \text{ is perpendicular to } BC \text{ and } CD$$

$$\int_C^D \vec{B} \cdot d\vec{l} = 0,$$

Since outside the solenoid the magnetic field lines are widely spaced hence there is a very weak field (practically zero)

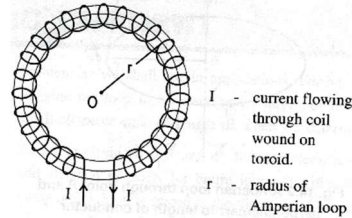
$$\text{Thus, } \int_A^B \vec{B} \cdot d\vec{l} = \int_A^B \vec{B} \cdot d\vec{l} = \int_A^B B dl \cos\theta = \int_A^B B dl \cos 0 = \int_A^B B dl$$

$$B \int_A^B dl = B \cdot L$$

Thus, $\oint \vec{B} \cdot d\vec{l} = \mu_0(nLI)$ becomes $B \cdot L = \mu_0(nLI)$. Therefore, $B = \mu_0(nI)$

Near the ends, $B = \mu_0(nI)/2$

MAGNETIC INDUCTION ALONG THE AXIS OF TOROID:



Toroid is a solenoid bent into a shape of a hollow donut.

Consider a Amperian loop of radius r .

According to Ampere's law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$, but here the total current flowing is NI

where N is the total number of turns. Thus, $\oint \vec{B} \cdot d\vec{l} = \mu_0 NI$. Now, \vec{B} and $d\vec{l}$ are in same direction. Thus $\oint \vec{B} \cdot d\vec{l} = B \cdot dl = B(2\pi r)$

$$\text{Hence, } B(2\pi r) = \mu_0 NI \quad \text{Thus, } B = \frac{\mu_0 NI}{2\pi r} = \mu_0 nI$$

where n = number of turns per unit length of toroid. = $\frac{N}{2\pi r}$

